Lec 6: 09/16/2013

Aesthetic Problems of the Biag-Bang Model:

The "standard Cosmological Model" can describe the universe and its evolution to the present time in very good agreement with observational data. It is based on the FRW metric obtained from general relativity. The description breaks down at a time $t < t_{planck} \simeq 5 \times 10^{-44}$ sec, which is defined as:

$$t_{planck} \equiv \sqrt{\frac{\hbar G}{c^5}}$$

This is the moment before which quantum gravitational effects were important. However, for $t \gg t_{planck}$, general relativity is valid and the hot big-bang model (described by the FRW metric) works well. The issue of the initial singularity (called big bang) aside, which is thought to be resolved by a theory of "quantum gravity", the big-bang model is theoretically consistent and its predictions are confirmed by observations.

Nevertheless, it suffers from some aesthetic problems. Here we are going to discuss these issues in more detail, in particular whether they can be addressed in a dynamical manner.

Flatness Problem:

As we saw, one can define a "critical density" for the universe according to:

$$\rho_{crit} \equiv \frac{3H^2}{8\pi G}$$

Normalizing both sides of the first Friedmann equation with ρ_{crit}, we have:

$$1 = \frac{\rho_{tot}}{\rho_{crit}} - \frac{k}{a^2\rho_{crit}}$$

We can define $\Omega \equiv \frac{\rho}{\rho_{crit}}$, which leads to:

$$\Omega_{tot} - 1 = \frac{k}{a^2\rho_{crit}}$$

Here ρ_{tot} denotes the total energy density in the universe,

$\rho_{tot} = \rho_m + \rho_r + \rho_\Lambda \Rightarrow \Omega_{tot} = \Omega_m + \Omega_r + \Omega_\Lambda$

If the universe is geometrically flat ($k=0$), then $\Omega = 1$.

Note that for a flat universe $\Omega = 1$ at all times. If the universe is open ($k=-1$) or closed ($k=+1$), then $\Omega < 1$ or $\Omega > 1$ respectively. In these cases $\Omega - 1$ evolves in time, and hence will not be a constant. We note that $\Omega - 1 \propto \frac{1}{a^2 H^2} = \frac{1}{\dot{a}^2}$.

Observations indicate that $|\Omega_0 - 1| \sim O(1\%)$, where Ω_0 is the value of Ω at the present time. We can use this to extrapolate $\Omega - 1$ back to very early times.

We note that:

$$\begin{cases} \Omega - 1 > 0 & k = +1 \\ \Omega - 1 < 0 & k = -1 \end{cases} \qquad (\mathrm{I})$$

$$\frac{d(\Omega-1)}{dt} \propto -k\frac{\ddot{a}}{\dot{a}^3}$$

In a decelerating universe $\ddot{a}<0$, which implies:

$$\begin{cases} \dfrac{d(\Omega-1)}{dt} > 0 & k=+1 \\ \dfrac{d(\Omega-1)}{dt} < 0 & k=-1 \end{cases} \qquad (\text{II})$$

Therefore, in a closed universe $\Omega-1$ becomes increasingly more positive, while in an open universe $\Omega-1$ becomes increasingly more negative, during decelerating expansion. This has been the case for our universe for the first $\sim 10^{10}$ years of its life. Then, the question is how close Ω must have been to 1 initially so that $|\Omega-1| \sim O(1\%)$ today. To see the level of the smallness of the initial value of $|\Omega-1|$, let us make the following estimate. The universe, in the hot big-bang scenario, was in a radiation-dominated phase for $t_{Planck} < t < t_{eq}$, where

$t_{eq} \simeq 50{,}000$ years. It entered a matter-dominated phase for $t_{eq} < t < t_{vac}$, where $t_{vac} \simeq 10^{10}$ years. During these phases we have:

$$t_{planck} < t < t_{eq} \Rightarrow a_{(t)} \propto t^{\frac{1}{2}},\ H \propto t^{-1} \Rightarrow a^2 H^2 \propto t^{-1} \Rightarrow |\Omega - 1| \propto t$$

$$t_{eq} < t < t_{vac} \Rightarrow a_{(t)} \propto t^{\frac{2}{3}},\ H \propto t^{-1} \Rightarrow a^2 H^2 \propto t^{-\frac{2}{3}} \Rightarrow |\Omega - 1| \propto t^{\frac{2}{3}}$$

As a result:

$$\frac{|\Omega - 1|_{eq}}{|\Omega - 1|_{planck}} \sim \frac{t_{eq}}{t_{planck}},\quad \frac{|\Omega - 1|_{vac}}{|\Omega - 1|_{eq}} \sim \left(\frac{t_{vac}}{t_{planck}}\right)^{\frac{2}{3}}$$

Putting these together, and using an approximation $\Omega_0 \sim \Omega_{vac}$, we find:

$$|\Omega - 1|_{planck} < O(10^{-60})$$

Therefore, the observational fact that the universe is very close to a geometrically flat one today, requiries that $|\Omega - 1|$ be extraordinarily small shortly after the big bang.

Note that $|\Omega - 1| \propto \frac{1}{a^2 H a}$, and hence:

$$a^2 > 10^{60} H^{-2} \qquad t = t_{planck}$$

This can be interpretted as that the universe had to be very large from the beginning in order to look almost flat today. It can be intuitively understood as follows. A geometrically open or closed universe looks flat locally. Today, we can see distances that are > 14 Gly, yet cannot notice if the universe is curved or not. This can be attributed to the fact that the universe was very large from the beginning (as compared with its natural size $\sim H^{-1}$), so even today what we can observe is only our local neighbourhood.

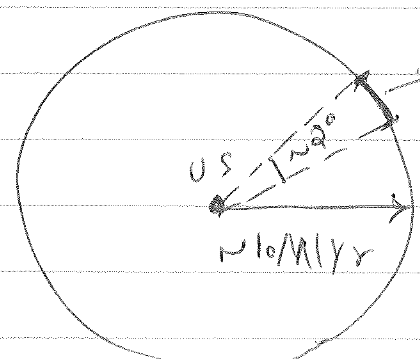
Isotropy Problem:

This has to do with the fact that the universe is so isotropic and homogenous at large scales. This is manifested most evidently in the CMB temperature, which is the same in all directions at the level of 1 in 10^5. The point is that not all the points in the sky were in causal contact at the epoch of recombination, $t_{rec} \sim 400{,}000$ years, when the CMB photons decoupled and started to move freely.

To see this, consider the furthest distance that we can see today, which is approximately 14 Glyr. At the time of recombination, this distance, which is the distance between us and the "surface of last scattering" where CMB photons started their travel, was smaller by an approximate factor of $\left(\frac{t_{rec}}{t_0}\right)^{2/3}$. We have ignored the late accelerated expansion of the universe between t_{vac} and t_0, but this does not affect the essence of our argument. The distance between us and the surface of last scattering therefore was ~ 10 Mlyr at $t = t_{rec}$.

On the other hand, the maximum distance that light had traveled until t_{rec} was $\sim 400{,}000$ lyr. This can be shown pictorially as follows:

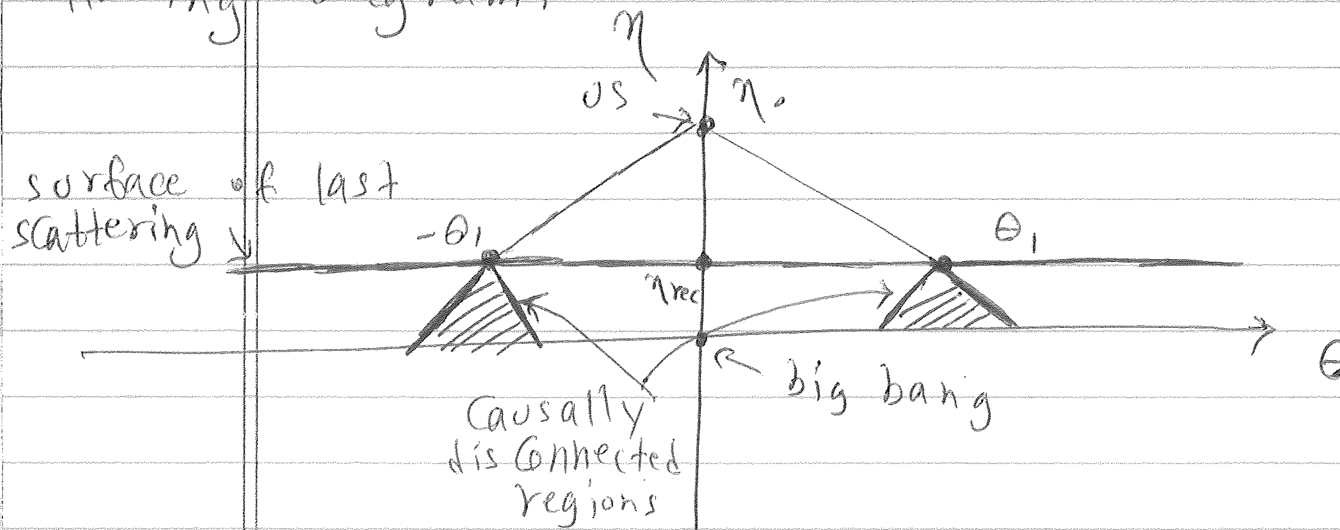
$t = t_{rec}$



surface of last scattering in the sky

It is seen that points in the sky that are apart by an angular distance $\lesssim 2°$ were in causal contact with each other at t_{rec}. The question is then how points with a much larger angular distance seem to have the same temperature. This is the so-called isotropy problem. It can also be seen by looking at the trajectory of CMB photons and the past light cone of points on the surface of last scattering. By switching to the conformal time η (where $dt \equiv a(t)\, d\eta$

the trajectory of light becomes a straight line $d\eta = dr$ just like in the special relativity. This leads to the following diagram:



Photons arriving from points at angles $\theta_1, -\theta_1$ have almost the same temperature. However, the past light cone of these photons do not overlap meaning that they could not have communicated with each othere between the big bang and the epoch of recombination. This leads to the isotropy problem, which is why they have essentially the same temperature.